Optimum Income Taxation and Layoff Taxes[*]

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Abstract

This paper analyzes optimum income taxation in a model with endogenous job destruction that gives rise to unemployment. It is shown that optimal tax schemes comprise both payroll and layoff taxes when the state provides public unemployment insurance and aims at redistributing income. The optimal layoff tax is equal to the *social cost of job destruction*, which amounts to the sum of unemployment benefits (that the state pays to unemployed workers) and payroll taxes (that the state does not get when workers are unemployed).

KEYWORDS: Layoff taxes, Optimal taxation, Job destruction.
JEL CODES: H21, H32, J38, J65

1 Introduction

So far, the influence of taxes on job creation and destruction has been neglected to a large extent by the economics of taxation (see Kaplov, 2006, Salanié, 2003, for recent surveys). Yet, many empirical studies have shown that modern economies face a dramatic process of job creation and job destruction that implies unemployment but that is also an important source of growth. In developed countries, the risk of unemployment induced by the process of job creation and job destruction is covered, to some extent, by public insurance. Moreover, in these countries, the government redistributes income from high income individuals towards low income individuals. Accordingly, public unemployment insurance interacts with income redistribution policies. Strickingly, there is no paper that has studied such an interaction. The aim of our paper is to analyze this issue.

In a seminal paper, Feldstein (1976) argued that payroll taxes used to finance unemployment benefits in most OECD countries induce too many layoffs, because employers do not take into account the cost of unemployment insurance provided by the state. To avoid this excess of job destruction unemployment insurance has to be financed by *layoff* taxes. The experience rating system used in the United States is an example of layoff taxes that induce firms to internalize the cost associated with their layoff decisions (Burdett and Wright, 1989a, b, Anderson and Meyer, 1993, 2000, Blanchard and Tirole, 2004, Cahuc and Malherbet, 2004).

In this paper, it is argued that layoff taxes are not only a natural counterpart to the state provision of unemployment benefits: they are also a natural counterpart to other public expenditures. Indeed, when employers destroy jobs, they do not take into account that workers who are fired will cost more to public finances. In this context, if individuals bring less in the budget of the state when they are unemployed than when they are employed, the *social value* of jobs, that is their value for the entire society, is larger than their *private value*, that is their value for the worker and the employer. This phenomenon can lead to excessive job destruction in the

absence of layoff taxes. Therefore, layoff taxes should not be only a part of the unemployment insurance system. They should also be integrated as an instrument in the overall tax system used to finance public expenditures.

We explore this idea in the standard approach of optimal taxation models (Mirrlees, 1971) which analyzes the tax-subsidy schemes that implement second-best allocations when the state has incomplete information about the characteristics of individuals. More precisely, we follow the approach of Diamond (1980) in which individuals, whose only decision is whether to work or not, differ in their taste for leisure as well in ability (see also: Beaudry and Blackorby, 1997, Choné and Laroque, 2005, Laroque, 2005, Saez, 2002). In Diamond's model, the ability of employees, which determines the market income and then the level of taxes, is observable, but taste for leisure is private information. In our paper, Diamond's model is enriched in order to account for unemployment and job destruction. It is assumed that the productivity of each job depends on the ability of the worker, that is common knowledge only when he participates in the labor market, and a random job specific productivity shock, that is privately known by the firm and the worker once the worker has been recruited. If the value of the specific productivity shock is too low, the job is destroyed and the individual becomes unemployed. Moreover, firms are risk neutral, workers are risk averse and it is assumed that unemployment insurance is provided by the state.[1]

Our paper analyzes the optimal tax-subsidy schemes that implement second-best allocations when there is endogenous job destruction. The main result is that optimal tax schemes comprise

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[1]The "implicit contract literature" has shown that risk neutral firms fully insure workers against income fluctuations by giving constant wages to the employees and unemployment benefits to the workers they layoff (Baily, 1974, Azariadis, 1975, Rosen 1985, Pissarides, 2001), However, in the real world, unemployment insurance is not provided by firms. Some rare exceptions are presented and discussed by Chui and Karni (1998) who stressed that the failure of the private sector to provide unemployment insurance can be explained by the interaction of adverse selection and moral hazard problems: an isolated firm that would offer private insurance would attract workers with strong work aversion, who would try to be fired as soon as they become eligible to the unemployment benefits. If work aversions are not observable and the level of effort of the employees not verifiable, it can be the case that private unemployment insurance cannot emerge. In our paper, we assume, like Burdett and Wright (1989a,b) and Blanchard and Tirole (2004) among many others, that unemployment insurance is provided by the state.

both payroll and layoff taxes when the state provides public unemployment insurance and aims at redistributing income. It turns out that the optimal layoff tax is equal to the social cost of job destruction, which amounts to the sum of the unemployment benefits (that the state pays to unemployed workers) and payroll taxes (that the state does not get when workers are unemployed). More precisely, if the state does not aim at redistributing income across individuals with different abilities, it is shown that layoff-taxes must be equal to unemployment benefits. This is a simple generalization of the result obtained by Blanchard and Tirole (2004) in a model without participation decision and without heterogeneity of workers. If the state aims at redistributing income across individuals with different abilities, first-best allocations cannot be reached because the taste for leisure is private information. The originality of our paper is to show that second-best allocations are obtained with layoff taxes equal to the sum of the unemployment benefits and the payroll tax in that case.

The paper is organized as follows. The preferences, the technology and the first-best allocations are presented in section 2. Section 3 is devoted to the analysis of the tax-subsidy schemes that allow the state to reach second-best allocations. Section 4 concludes.

2 The model

2.1 Preferences and technology

We consider a static economy with a continuum of individuals whose size is normalized to one. There are two goods: labor and a marketable good produced thanks to labor. Individuals outside the labor force do not produce the marketable good. Individuals inside the labor force can be either employed or unemployed.

An individual is described by a set of exogenous characteristics, denoted by $s = (y, z)$; where y stands for his ability and z for his taste for leisure. We assume that (y, z) has a joint density $h(y, z)$ with $h > 0$ over the support $S \subset [y_{\min}, +\infty) \times \mathbb{R}$, h is continuous and $y_{\min} \geq 0$.

The preferences of the type-s individual are represented by the utility function $v(c + z\ell)$, twice derivable, increasing and strictly concave, where $c \geq 0$ denotes consumption, $\ell \in \{0, 1\}$ denotes leisure that amounts to zero if the individuals is active (either employed or unemployed) and to one if he is not in the labor force.[2] The set of inactive individuals is denoted by S_I and the set of active agents (which comprises employed and unemployed workers) is denoted by S_A.

Creating a job for a type-(y, z) individual entails a fixed cost represented by a strictly positive and continuous function $k(y)$. When a type-(y, z) individual gets a job, he can produce $x \cdot y$ units of the marketable good, where $x \in \mathbb{R}$ is an idiosyncratic shock drawn in a distribution with a continuous differentiable cumulative distribution function denoted by G. The average of x is finite. Each individual can occupy at most one job.

An allocation defines the consumption and the employment status of all the agents of the economy. It is a mapping that associates to each type-s individual, conditional on the realization of the productivity shock x for active individuals, his consumption: $c(s)$, if $s \in S_I$, $c(s, x)$ if $s \in S_A$, and his employment status: inactive ($\ell(s) = 1$), employed ($\ell(s) = 0$ and marketable production $= x \cdot y$) or unemployed ($\ell(s) = 0$ and marketable production $= 0$). The allocation of individuals between employment and unemployment amounts to defining the set of values of the productivity parameter x, denoted by $W(y) \subset \mathbb{R}$, for which the type-(y, z) individuals who belong to S_A work.

All allocations have to satisfy the feasibility constraint:

$$\int_{S_A} \left[Y(y) - \int_{\mathbb{R}} c(y, z, x) dG(x) \right] h(y, z) dy dz = \int_{S_I} c(y, z) h(y, z) dy dz, \qquad (1)$$

where $Y(y) = y \int_{W(y)} x dG(x) - k(y)$ stands for the average net production of employees with

ability y.

2.2 First-best allocations

First-best allocations are chosen by a fully informed planner who has complete information on the pair $s = (y, z)$ describing each agent's characteristics and on the productivity shocks x. First-best allocations are such that there are no other feasible allocations that can improve the welfare of at least one agent without worsening the welfare of the others. It is assumed that feasible allocations are ranked according to the expected utility criterion conditional on characteristics (y, z) and on the realization of the productivity shocks x. The time sequence of events that describes the decision of the planner runs as follows:

1) The planner decides which set of agents $s \in S_I$ are inactive ($\ell(s) = 1$), and which set of agents $s \in S_A$ are allowed to participate in the production of the consumption good ($\ell(s) = 0$). It costs $k(y)$ to assign a type-(y, z) worker to S_A. The planner also announces the consumption of the marketable good of every type-s individuals. The consumption can be conditional on the realization of the productivity shocks for active individuals.

2) Every individual in S_A makes a draw x from the cdf function G that allows his potential production to reach the level $x \cdot y$. After observing x, the planner decides whether each individual in S_A actually produces or not (this is the job destruction decision). Individuals produce and consume according to the plan announced at step **1)**.

The first-best allocations can be obtained by backward induction.

At step **2)**, once x has been drawn, it is worthwhile keeping employed the individuals who produce more on-the-job than in unemployment. The marketable production of a type-(y, z) individual amounts to y. Therefore, it is worthwhile keeping employed the type-(y, z) workers such that $x \cdot y \geq 0$. The choice of the set of values of the productivity parameter x, denoted by $W(y) \subset \mathbb{R}$, for which the type-(y, z) individuals who belong to S_A work, boils down to the choice of the reservation productivity below which the type-(y, z) individuals belonging to S_A

are unemployed. The first-best reservation productivity, denoted by X^*, satisfies the *productive efficiency condition*:

$$X^* = 0. \tag{2}$$

It turns out that job separation is efficient only if realized productivity is negative. This result relies on the assumption that the disutility associated with unemployment is the same as the disutility associated with work. This assumption, which is chosen for the sake of simplicity, will be relaxed below (in section 3.3.). If it was supposed that the disutility of unemployment was lower than the disutility of work, jobs with positive productivity would be destroyed.

Since it costs $k(y)$ to assign the type-(y, z) individual to S_A, the average net first-best production of an individual with ability y belonging to S_A amounts to $Y^*(y) = -k(y) + y \int_0^{+\infty} x dG(x)$.

At step **1)**, the planner chooses the set of individuals who participate in the marketable activities. It can easily be understood that the set S_A of active agents only comprises type-s individuals such that $Y^*(y) \geq z$. Imagine that we can find in S_A an agent with $Y^*(y) < z$. This agent can get the same utility level when he is inactive if his consumption of the marketable good is decreased by z. This allows the social planner to win z and lose $Y^*(y)$ as forgone production, which yields a positive net gain equal to $z - Y^*(y)$. Thus, it is not optimal to have an active individual whose taste for leisure is larger that his expected production. An analogous reasoning shows that the set S_I comprises type-s individuals such that $Y^*(y) < z$. In other words, the participation decision reads:

$$\ell^*(y, z) = \begin{cases} 0 \text{ if } z \leq Y^*(y) = -k(y) + y \int_0^{+\infty} x dG(x) \\ 1 \text{ otherwise.} \end{cases} \tag{3}$$

At step **1)** the planner has also to choose the consumption of the marketable good for every individual. For the active individuals, the assumption of risk aversion implies that the certainty equivalent income of the lottery $\{c(s, x)\}$ is smaller than the expected consumption $\int_{\mathbb{R}} c(s, x) dG(x)$. Accordingly, a social planner whose decisions are based upon the expected

utility criterion can always save resources by providing to the type-s individuals belonging to S_A the certainty equivalent associated with the lottery $\{c(s,x)\}$. It follows that the first-best allocations necessarily insure all individuals in S_A against productivity shocks and give them the same consumption whether employed or unemployed and whatever the realization of x.

The properties of the first-best allocations are summarized in the following proposition:

Proposition 1 *A feasible allocation is a first-best allocation if and only if:*

1. *Active individuals are employed when $x \geq 0$ and unemployed otherwise.*

2. *Every agent with the same type s belonging to the set S_A of active individuals gets the same consumption level whatever the value of x.*

3. *The set S_A of active individuals comprises all the type-s individuals such that $Y^*(y) \geq z$, and the set S_I of inactive individuals comprises all the agents such that $Y^*(y) < z$.*

Proof. see appendix A. ■

Claim *3* of proposition 1 is a particular case of a more general result established in Laroque (2005) stating that in an economy with labor supply choice at the extensive margin, where the agents choose whether to work or not to work, it is Pareto optimal that someone works if and only if his productivity is larger than the extra necessary income to compensate him for the hardship of work. In our economy, the agents are perfectly insured against unemployment risks and the extra necessary income to compensate an individual, with taste for leisure z, for being active is simply equal to[3] z while his expected productivity amounts to $Y^*(y)$.

3 Second-best allocations and optimal tax-subsidy schemes

This section is devoted to the design of optimal fiscal policies in a framework in which the state is committed to a tax-subsidy scheme and where the marketable good is produced on a perfectly

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[3]It should be noticed that nothing prevents the state to give different consumption levels to agents with the same ability y but with different z if the state exhibits preferences compatible with such an allocation.

competitive market. Contrary to the first-best environment, the state does not observe the characteristics of the agents. Namely, the taste for leisure z always remains private information of the worker. The ability y and the idiosyncratic productivity shock x are observed by the firm and the worker but are not verifiable. The distributions of x, y and z are common knowledge. The state only observes the labor contracts and whether individuals work. This implies that the state knows who has been fired and is able to distinguish unemployed workers from inactive individuals. In this situation, the tax-subsidy scheme can only depend on the elements of the labor contracts and on the employment status (employed, unemployed, inactive).

First, the decentralized equilibrium is studied. Then, we analyze the optimal policies.

3.1   Decentralized equilibrium

The marketable good is produced by firms on a competitive market with free entry. As the labor contracts only stipulate wages, the state can use tax-subsidy schemes conditional on three elements only: 1) the wage, denoted by w; 2) the employment status (employed, unemployed or inactive); 3) the job destruction decision. We consider tax-subsidy schemes that comprise a payroll tax,[4] $\tau(w)$, a layoff tax, $f(w)$, unemployment benefits, $b(w)$, and an income guarantee $\rho \geq 0$ paid to the inactive persons. It should be noticed that the set of instruments used in the literature on optimal taxation where, contrary to our model, the productivity of each individual only depends on his ability but is not influenced by a random term, comprises an income tax (or equivalently a payroll tax) and an income garantee. In our paper, the presence of shocks on productivity implies that it is necessary to expand the set of instruments that the government needs to achieve efficient allocations.[5]

The overall consumption of the individual who has signed a labor contract that stipulates

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[4]The payroll tax can equivalently be considered as an income tax. Actually $\tau(w)$ stands for the tax wedge that can be paid indifferently by the employer or by the employee.

[5]The government can exploit the correlation between observable individual characteristics (such as age, sex, years of education...) and abilities to screen individuals with taxes that depends on these observable characteristics. This is an interesting issue that is beyond the scope of this paper.

a wage w amounts to w if he is employed and to $b(w)$ if he is unemployed. The type-(y, z) individual that does not participate in the labor market gets a utility level $v(z + \rho)$. The wage w entails labor costs equal to $w + \tau(w)$ if workers remain employed and to $f(w)$ if they are fired. When a job is destroyed, no output is produced. However, it is assumed that firms are always able to pay layoff taxes even when their jobs are unproductive. This means that there is a perfect financial market that allows firms to fully diversify their risk. We will discuss at the end of this section the case where there is a limit to what the state can actually collect because firms cannot get complete insurance.

The time sequence of events runs as follows:

1) The state announces a balanced budget tax-subsidy scheme $\{\tau(w), f(w), b(w), \rho\}$.

2) Individuals decide whether they belong to the labor force or stay inactive.[6]

3) Employers create jobs and enter into Bertrand competition to hire workers.[7]

4) The specific productivity shocks x occur and employers decide whether they keep the workers or they destroy the jobs. Then, employers pay the wage and the payroll tax for every continuing job. Every destroyed job gives rise to the payment of layoff taxes. Employed workers get a wage w, unemployed workers get unemployment benefits $b(w)$ and inactive individuals get the garantee income ρ.

In this subsection, we characterize the existence and the properties of the competitive equilibrium of the labor market for the tax-subsidy scheme announced at step **1)**. This problem is solved by backward induction.

At step **4)** firms destroy jobs if and only if their profits, $x \cdot y - w - \tau(w)$, are lower than their destruction costs, $-f(w)$. The job destruction decision boils down to the choice of a reservation

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[6]It is assumed that individuals who decide to belong to the labor force reveal their true productivity. If they had the possibility to behave as agents of lower productivity without cost, truthful revelation would only obtain under the condition that their wage w be non decreasing in y, a condition that will be satisfied for second-best allocations.

[7]There are at least as many potential jobs as there are potential active individuals.

value of the productivity parameter x, denoted by $X(w, y)$, below which job are destroyed.[8] The reservation productivity reads:

$$X(w, y) = [w + \tau(w) - f(w)] / y. \tag{4}$$

For individuals with ability y, the job destruction rate (or equivalently the unemployment rate), denoted by $q(w, y)$, is equal to $G(X(w, y))$.

At step **3)**, the expected profit of an employer offering a contract w to a type-(y, z) worker, denoted by $J(w, y)$, reads[9]

$$J(w, y) = -k(y) + \int_{X(w,y)}^{+\infty} [x \cdot y - w - \tau(w)] \, dG(x) - q(w, y)f(w). \tag{5}$$

Existence and unicity of the Bertrand equilibrium depend on the properties of the functions $k(\cdot)$ and $G(\cdot)$ and of the functions $\tau(\cdot)$, $b(\cdot)$, $f(\cdot)$ describing the tax-subsidy schedule. We shall assume that all these functions are such that the expected profit $J(w, y)$ satisfies the properties summarized in Assumption 1:

Assumption 1

1.i) $\forall y, \ \{w \geq 0 \mid J(w, y) = 0\} \neq \emptyset$.

1.ii) $\forall y, \ \lim_{w \to +\infty} J(w, y) < 0$.

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[8] This behavior comes from the assumption that the reservation value of the productivity parameter x is not contractable and that the firm cannot commit ex-ante to this reservation value by keeping aside funds to be paid to a third party in case of layoff. See the discussion in Blanchard and Tirole (2004).

[9] It can easily be seen that the introduction of an operating tax that depends on the wage on the top of the payroll tax and the layoff tax would not be useful. To show that we have enough instruments with the payroll tax and the layoff tax, imagine that there exists an additionnal operating, denoted by $T(w)$, which can be viewed as an additionnal cost of creating a job depending upon the wage offered. Simple calculations enable to write the expected profit as:

$$J(w, y) = -k(y) - T(w) - \tau(w) + \int_{X(w,y)}^{+\infty} [x \cdot y - w] \, dG(x) - q(w, y) [f(w) - \tau(w)]$$

Since $X(w, y)$ only depends on the difference $f(w) - \tau(w)$, it appears that the expected profit only depends on two (new) instruments $\tau_1(w)$ and $f_1(w)$ defined by $\tau_1(w) = T(w) + \tau(w)$ and $f_1(w) = f(w) - \tau(w)$. Therefore, the sets of instruments $(T(w), f(w))$ and $(\tau(w), f(w))$ are equivalent.

Conditions *1.i)* and *1.ii)* are necessary and sufficient conditions to obtain a unique Bertrand equilibrium with positive finite wages.[10] More precisely, condition *1.ii)* prevents employers from offering unbounded wages, thus Bertrand competition between the employers drives expected profits to zero and condition *1.i)* states that for any y there exists at least one positive wage giving an expected profit equal to zero.[11] Then the Bertrand equilibrium wage is the highest value of w that solves the zero profit condition $J(w, y) = 0$. In other words, conditions *1.i)* and *1.ii)* imply that there exists a unique equilibrium wage contract $w(y)$ offered to the type-(y, z) workers, which reads:

$$w(y) = \sup \{w \geq 0\} \mid J(w, y) = 0\}. \tag{6}$$

Furthermore, we can obtain a precise result concerning the monotonicity of the equilibrium wage function $w(y)$ if we add the following (reasonable) assumption.

Assumption 2

2.i) $\forall y$, $J(w, y)$ is continuous in w.

2.ii) $\forall w$, $J(w, y)$ is strictly increasing with respect to y.

Proposition 2 *When Assumptions 1 and 2 are satisfied there exists a unique equilibrium wage function $w(y)$ which is strictly increasing with respect to the ability level y.*

Proof. According to *2.ii)*, for any $y' > y$ the Bertrand equilibrium wage $w(y)$ corresponding to the ability level y satisfies $J(w(y), y') > 0$. The limit condition *1.ii)* and the continuity condition *2.i)* then imply that there exists (at least) one finite wage $w > w(y)$ such that $J(w, y') = 0$. Finally, the definition (6) of the Bertrand equilibrium wage entails that the Bertrand equilibrium wage $w(y')$ corresponding to the ability $y' > y$ is such that $w(y') \geq w > w(y)$. ■

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[10] In our model the possibility of negative wages is ruled out because the agents cannot borrow and do not have any initial resources. Thus, they cannot pay for having a job.

[11] It can also be checked that the positive fixed cost to creating a job $k(y) > 0$ ensures that the reservation productivity $X(w, y)$ is finite when the free entry condition $J(w, y) = 0$ is fulfilled and when the layoff and the payroll taxes are equal to zero (this can be easily seen from equations (4) and (5)).

Proposition 2 merely shows that there exists a unique strictly increasing wage function in equilibrium if the profit function satisfies properties which are defined in Assumptions 1 and 2. The set of second-best optimal subsidy schemes which imply profit functions that do satisfy Assumptions 1 and 2 will be precisely defined below. The definition of the second-best optimal subsidy schemes will also allow us to describe more precisely the properties of the second-best wage functions.

At step **2)**, the type-(y, z) individuals decide to enter into the labor market if and only if the participation constraint[12]

$$[1 - q(w, y)] v(w) + q(w, y)v [b(w)] \geq v(z + \rho)$$

is fulfilled. This condition implies that only individuals whose taste for leisure z is smaller than the threshold value, $Z(w, y)$, defined by

$$v [Z(w, y) + \rho] = [1 - q(w, y)] v(w) + q(w, y)v [b(w)], \tag{7}$$

belong to the labor force. $Z(w, y)$ can be interpreted as the *financial incentives to work* provided to type-(y, z) individuals.

In other words, the participation decision for a type-(y, z) worker receiving a wage offer w reads:

$$\ell(y, z) = \begin{cases} 0 & \text{if} \quad z \leq Z(w, y) \\ 1 & \text{if} \quad z > Z(w, y). \end{cases} \tag{8}$$

Eventually, given any tax-subsidy scheme $\{\tau(w), f(w), b(w), \rho\}$ that satisfies Assumptions 1 and 2, there exists a single decentralized equilibrium that defines an allocation entirely characterized by three functions of y : the wage $w(y)$ (equation (6)) which accrues to type-(y, z) employees, the financial incentives to work $Z(w(y), y)$ (equation (7)) and the reservation productivity $X(w(y), y)$ (equation (4)) below which jobs are destroyed.

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[12]It should be noted that when Proposition 2 is satisfied, any active worker has interest to reveal his true ability even if he can understate his ability at no cost, since the net wage is a strictly increasing function of productivity.

13

3.2 Optimal tax-subsidy schemes

At decentralized equilibrium, the welfare of each individual is influenced by the tax-subsidy scheme chosen by the state. We shall use a Pareto criterion to define the optimal policies. By definition, a tax-subsidy scheme is optimal if it is *feasible – i.e.* satisfies the budget constraint of the state[13]– and if there is no other feasible tax-subsidy scheme that can improve the welfare of at least one agent without worsening the welfare of the others. In other words, optimal tax-subsidy schemes implement second-best allocations. Like in the first-best environment, second-best allocations must satisfy efficiency conditions concerning the insurance against unemployment risk, the job destruction decisions, and the choice between activity and inactivity. In the sequel, we characterize more intuitively than rigorously the properties of second-best allocations. A formal proof of all these properties is given in Appendix B.

Insurance

It can easily be understood that efficiency requires that the state, which provides the unemployment benefits $b(w)$, must insure the active agents against unemployment risks. The expected utility of an agent who accepts a contract offering a wage w amounts to $(1-q)v(w) + qv(b(w))$. Risk aversion implies that the certainty equivalent income of the lottery $\{w, b(w), q\}$ is smaller than the expected consumption $(1-q)w + qb(w)$. Therefore, the state can always save resources by designing a tax-subsidy scheme that provides to any active agent the certainty equivalent of his income whether he is employed or unemployed. Hence, any optimal policy satisfies:

$$b(w) = w. \tag{9}$$

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[13]Let us posit $b(y) = b(w(y))$, $\tau(y) = \tau(w(y))$, $f(y) = f(w(y))$, $X(y) = X(w(y), y)$, $q(y) = G(X(y))$ and $Z(y) = Z(w(y), y)$. The budget constraint of the state reads:

$$\int_{y_{\min}}^{+\infty} \left(\int_{-\infty}^{Z(y)} \{[1 - G(X(y))]\,\tau(y) + G(X(y))\,[f(y) - b(y)]\}\,h(y,z)dz \right) dy \geq \rho \left[1 - \int_{y_{\min}}^{+\infty} H\left[y, Z(y)\right] dy \right]$$

where $H\left[y, Z(y)\right] = \int_{-\infty}^{Z(y)} h(y,z)dz$.

Job destruction

Alike what happens in the first-best environment, efficiency requires the productive efficiency condition (2) to be satisfied. The reason is the same as in the first-best environment: As individuals are fully insured against the unemployment risk, they get the same utility level whether employed or unemployed, but the overall production is larger when jobs producing $x \cdot y < 0$ are destroyed (because the production of an unemployed is equal to 0).[14] Looking at the market value of the reservation productivity given by equation (4), it follows that any optimal policy has to satisfy:

$$w + \tau(w) - f(w) = 0. \tag{10}$$

In such circumstances, one has $X(w(y), y) = X^* = 0$. The equilibrium job destruction rate is equal to $q(w(y), y) = G(0)$, and the average net production of an active individual of ability y is worth $Y^*(y) = -k(y) + y \int_0^{+\infty} x dG(x)$.

Participation decisions

The efficiency requirement on participation decisions amounts to impose constraints on the financial incentives to work $Z(w(y), y)$ that will be denoted as $Z(y)$. The simple idea here, put to the fore by Laroque (2005), is that feasible financial incentives to work $Z(y)$, such that $Z(y) \leq Y^*(y)$,[15] support a second-best allocation if and only if no category of ability y is overtaxed at $Z(y)$, i.e. if and only if it is not possible to increase the incentive to work and the amount of net taxes collected by the state. The intuition for this result is straightforward: a situation in which it is possible to increase both the welfare of the workers of ability y and the

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[14]If employed and unemployed workers did not have the same opportunity cost of participating in the labor market jobs with positive productivity could be destroyed. See the discussion section below and Appendix C.

[15]Following Laroque, we restrict the analysis of necessary and sufficient conditions to tax-subsidy schemes such that $Z(y) \leq Y^*(y)$. Looking at more general tax-subsidy schemes is interesting but is not central to our analysis. Since $\tau(w(y)) = Y^*(y) - Z(y) - \rho$ and $\rho \geq 0$, the assumption $Z(y) \leq Y^*(y)$ is equivalent to $\tau(w) + \rho \geq 0$. This condition simply states that net taxes payed by active individuals have to be positive. It should be noticed that the condition $\tau(w) + \rho \geq 0$ is compatible with negative income tax such as the earned income tax credit in the US for instance.

total amount of tax they pay cannot be optimal. Such an allocation is on the wrong side of the Laffer curve.

It is also important to remark that the incentive to work has to be a strictly increasing function of the abiltiy. Equations (7) and (9) imply that $Z(y) = w(y) - \rho$. Since truthful revelation of the agents' productivity requires that the equilibrium wage $w(y)$ is a strictly increasing function of y, it follows that $Z(y)$ must be strictly increasing with y.

Let us denote by $\tilde{Z}(y)$ any strictly increasing function that belongs to the set of second-best financial incentives to work and such that $\tilde{Z}(y) \leq Y^*(y)$. Obviously, the shape of this function depends on the preference for redistribution of the state. If the aim of the state is to achieve the *laissez-faire* with no redistribution of income between individuals of different types, the state chooses $\tilde{Z}(y) = Y^*(y)$. In the opposite extreme case where the state is Rawlsian, i.e. maximizes the expected utility of the individuals in the worst situation, the shape of $\tilde{Z}(y)$ is chosen to maximize the income guarantee ρ. The properties of the optimal tax-subsidy schemes are summarized in the following proposition.

Proposition 3 *For any strictly increasing function $\tilde{Z}(y)$ that belongs to the set of second-best financial incentives to work and such that $\tilde{Z}(y) \leq Y^*(y)$,*

A) a feasible tax-subsidy scheme $\{\tau(w), f(w), b(w), \rho\}$ such that $\tau(w) + \rho \geq 0$ is optimal if and only if:

1. *$b(w) = w$*

2. *$f(w) = b(w) + \tau(w)$*

3. *The layoff tax schedule $f(w)$ is defined by:*

$$f(w) = Y^* \left[\tilde{Z}^{-1}(w - \rho) \right] \quad with \quad \rho = \int_{y_{\min}}^{+\infty} \left[Y^*(y) - \tilde{Z}(y) \right] H \left[y, \tilde{Z}(y) \right] dy$$

B) Moreover, if $\tilde{Z}(y)$ is continuous and bounded and $Y^*(y)$ is strictly increasing, then Assumptions 1 and 2 are satisfied and therefore there exists a unique decentralized competitive equilibrium associated with the optimal tax-subsidy scheme.

Proof. *see appendix B.* ∎

This proposition shows that second-best optimal policies necessarily include layoff taxes. The result that the lay-off tax is needed to induce production efficiency is not particularly surprising. It is worth noting that it is not the fact that individuals (and firms) have private information about individual characteristics that gives rise to the optimal use of a lay-off tax but rather the simple fact that firms make the firing decisions. In the first-best, the planner makes the firing decisions. In the second-best firms make these decisions. As firms must pay workers positive wages, firms will fire too many workers. By imposing a tax on firms for firing a worker, the government can ensure that workers are only fired if they produce a negative amount of output.

More precisely, condition *2.* of Proposition 3 states that the optimal tax-subsidy schemes comprise layoff taxes that cover the *social cost* of job destructions, which amounts to the sum of the unemployment benefits and the payroll tax.[16] In other words, the social cost of job destructions is equal to the loss imposed to the state, which comprises the unemployment benefits, $b(w)$, that are obtained by the unemployed worker, but not by the employee, plus the payroll tax, $\tau(w)$, that is payed when the job is filled, but not any more when it is destroyed.

From this point of view, it is worth stressing that the social cost of job destruction only amounts to the unemployment benefits when the aim of the state is to provide insurance to active individuals without cross-subsidization among individuals with different types s. This case is characterized by the following Corollary:

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[16]The social cost of job destruction depends on the instruments used by the government. In our model we only consider the case where the government uses all the useful instruments at its disposal as it is the case in the optimal taxation approach.

Corollary 1 *The single first-best allocation attainable through the market allocation is implemented by the following tax-subsidy scheme:*

$$f(w) = b(w) = w, \rho = \tau(w) = 0 \tag{11}$$

The allocation is characterized by:

$$c(s) = \begin{cases} Y^*(y) & \text{if } z \leq Y^*(y) \\ 0 & \text{otherwise} \end{cases} \quad , \quad \ell(s) = \begin{cases} 0 & \text{if } z \leq Y^*(y) \\ 1 & \text{otherwise} \end{cases}$$

Proof. Condition *1* of Proposition 1 is satistied if $b(w) = w$. Equations (2) and (4) imply that condition *2* of Proposition 1 is satisfied if $f(w) = b(w)$ and $\tau(w) = 0$. According to condition *3* of Proposition 1, a first-best allocation requires that $Z(y) = Y^*(y)$. According to equation (7), one gets $Z(y) = w(y) = b(w(y))$ if $\rho = 0$ and $b(w) = w$. Then, equations (5) and (6) imply that $Z(y) = Y^*(y)$ if $Z(y) = w(y) = b(w(y)) = f(w(y))$ and $\tau(w) = 0$. ∎

Corollary 1 indicates that, in the first-best, unemployment benefits should not be financed by income taxes because there is no cross-subsidization across different types-s individuals. Unemployment benefits should be financed by layoff taxes only. Accordingly layoff taxes are needed to implement the first-best allocation without redistribution of income across individuals with different types s. When there is no cross-subsidization among individuals with different types s, every type-(y, z) individual gets the amount of marketable good that corresponds to his expected production, $Y^*(y)$, when he participates in the labor market and zero otherwise. In other words, the financial incentives to work take their maximum value: $Z(y) = Y^*(y)$. This implies that the decentralized equilibrium yields a first-best allocation. This situation, which corresponds to *laissez-faire*, arises when the government displays no preference for redistribution. When the government has some redistributive purposes towards individuals with low potential income, the income guarantee ρ becomes strictly positive and the wage $w(y)$ is merely equal to the second best incentive to work $\tilde{Z}(y)$ plus the income garantee ρ. The need to finance the income guarantee implies positive income taxes for some employees. The tax paid by employees

18

of ability y, which is equal to $Y^*(y) - w(y)$, depends on the preferences of the government and on the properties of the distributions of exogenous characteristics (y, z). The layoff tax paid when an employee with ability y is fired is equal to the average net production of employees with ability y (denoted by $Y^*(y)$).

Corollary 1 generalizes the result of Blanchard and Tirole (2004) – obtained in a framework with a single type s – according to which efficiency requires that layoff taxes be equal to unemployment benefits. Our approach, that takes into account the heterogeneity of individuals in the tradition of Mirrlees (1971), allows us to analyze how layoff taxes should be integrated in optimal tax-subsidy schemes when there is redistribution of income across individuals with different types in the presence of endogenous job destruction.

From this point of view, it is worth noting that Proposition 3 implies that the layoff tax is necessarily larger than unemployment benefits for at least some type-(y, z) workers, because positive $\tau(w)$ are needed for at least some type-(y, z) when there is a positive income guarantee ρ or redistribution of income across individuals with different types. It is the presence of payroll taxes that distorts the participation decisions. The negative impact of income taxation on labor supply is at the basis of the problem tackled by the research on optimal taxation *à la* Mirrlees (1971) and Diamond (1980) in which the state faces a trade-off between the degree of redistribution of income and the degree of participation in the labor force. When job destruction decisions are taken into account, layoff taxes belong very naturally to any optimal tax-subsidy scheme. In other words, layoff taxes are not only useful to finance unemployment benefits, as it is usually acknowledged, they are also useful to induce individuals to internalize the impact of their job destructions decisions on the budget of the state when there is income redistribution across individuals with different abilities y and different tastes for leisure.

3.3 Discussion

It should be noticed that the properties of the layoff tax defined in proposition 3 rely on specific assumptions that are worth discussing.

First, it is assumed that firms can always pay layoff tax because they can borrow on a perfect financial market. This assumption is surely too strong. Even in the absence of aggregate risk, the owners of many firms, especially small ones, are not fully diversified, and thus likely to act as if they were risk averse. And, even if entrepreneurs are risk neutral, information problems in financial markets are likely to lead to restrictions on the funds available to firms. Blanchard and Tirole (2004) have focused on the implications of limited funds. In that case, it appears that it is optimal to fully insure workers but the layoff tax is however reduced by the lack of funds that limits the ability of firms to pay layoff taxes. The same result can be derived in our framework: the amount of layoff tax would be lower than when employers can fully diversify their risk on perfect financial markets but it would still be determined by the preference for income redistribution of the state.

Second, we make the extreme assumption that individuals who are laid off (the unemployed workers) and individuals who are working have zero time devoted to leisure. If it was not the case, the state would not be able to fully insure the workers because the value from leisure time is not observed by the state. In order to understand what is going on in that case, let us consider the extreme assumption where individuals value their leisure time equally when they are laid off and when they are inactive. It turns out that type-(y, z) inactive and unemployed individuals reach the same level of utility when they get the same income. Therefore, it is optimal to give the same income to unemployed and to inactive individuals, so that $b(w) = \rho$. Only workers whose taste for leisure is smaller than the wage net of the income garantee participate in the labor market (i.e. $z < w(y) - \rho$). Accordingly, unemployed individuals are not fully insured and employed individuals are better off than unemployed workers. It can also be shown that the

layoff tax, which is equal to $b(w) + \tau(w)$, still depends on the preference for income redistribution of the state (see Appendix C).

Third, we assume that the government is able to distinguish between inactive and laid off workers. If it was not the case, inactive workers could claim that they have been fired to get unemployment benefits. This phenomenon implies that the state cannot any more fully insure unemployed workers. In that case, as shown by Blanchard and Tirole (2004), unemployment benefits have to be smaller than the wage, but it is still usefull to use layoff taxes that depend on unemployment benefits and on the degree of redistribution of income chosen by the state.

These few remarks suggest that the result according to which layoff taxes should be integrated in optimal tax-subsidy schemes is robust. This is not suprising to the extent that the social cost of job destruction depends on the degree of redistribution of income across individuals with different abilities and different tastes for leisure.

4   Conclusion

This paper shows that optimal tax-subsidy schemes should comprise layoff taxes. It turns out that optimal layoff taxes are linked to the intensity of the redistribution of income: the optimal layoff tax is equal to the social cost of job destruction, which amounts to the unemployment benefits paid to the fired worker plus the payroll taxes (used to redistribute income across individuals with different abilities or different tastes for leisure) that the state losses when the job is destroyed. Accordingly, layoff taxes should represent a larger share of the wage when there are higher payroll taxes due to a more intensive redistribution of income.

Although we think that our result according to which optimal tax-subsidy schemes should comprise layoff taxes is general and relevant, our analysis needs to be further developed in some directions. First, our framework takes into account only some externalities induced by job destruction decisions. Search and matching models stress that job destructions induce search

externalities which imply that the decentralized equilibrium does not yield *enough* job destructions (Aghion and Howitt, 1998, Mortensen and Pissarides, 1999). From this perspective, it would be worth introducing *negative* layoff taxes (Cahuc and Zylberberg, 2004, chapter 10). We need to know more on the interactions between externalities and on their relative magnitude to know the optimal level of layoff taxes. Second, our framework assumes a very simple form of labor contracts, without ex-post bargaining that gives rise to hold-up problems. Asymmetric information problems linked to unemployment insurance have also been neglected. Such issues are worth studying. Third, it is also important to look at the consequence of layoff tax in a dynamic model where the durations of employment and of unemployment can be influenced by firing costs.[17] These developments are on our research agenda.

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[17]This issue is explored, to some extent, in Cahuc and Zylberberg (2005).

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Appendix

A   Proof of Proposition 1

Necessary conditions have been shown in the text. It remains to be proved that any feasible allocation which satisfies conditions *1, 2* and *3* is Pareto optimal. Let us show that an allocation that makes every agent as least as well off and some strictly better off than an allocation which satisfies conditions *1, 2* and *3* is not feasible.

The feasibility constraint for a first-best allocation which satisfies conditions *1, 2* and *3* and yields consumptions denoted by $c(y,z)$ reads

$$\int_{S_A} Y^*(y)h(y,z)dydz = \int_{S_A} c(y,z)h(y,z)dydz + \int_{S_I} c(y,z)h(y,z)dydz,$$

where $Y^*(y) = y\int_0^{+\infty} xdG(x) - k(y)$ stands for the average first-best net production of employees with ability y.

Let us denote by $\hat{c}(y,z,x)$ the consumption of type-(y,z) active individuals and by $\hat{c}(y,z)$ the consumption of type-(y,z) inactive individuals for a feasible allocation (called henceforth the alternative allocation) that makes every individual at least as well off and some strictly better off than a first-best allocation (called henceforth the initial allocation) which satisfies conditions *1, 2* and *3* and yields consumptions denoted by $c(y,z)$.

The feasibility constraint for the alternative allocation reads

$$\int_{S_{\hat{A}}} \hat{Y}(y)h(y,z)dydz = \int_{S_{\hat{A}}} \left(\int_{\mathbb{R}} \hat{c}(y,z,x)dG(x)\right) h(y,z)dydz + \int_{S_{\hat{I}}} \hat{c}(y,z)h(y,z)dydz,$$

where $\hat{Y}(y) = y\int_{\hat{W}(y)} xdG(x) - k(y)$ stands for the average net production of employees with ability y and $S_{\hat{A}}$, $S_{\hat{I}}$ denote the set of active and inactive individuals respectively.

Let us denote by $S_{A\hat{A}}$ the set of agents who are active in both allocations, by $S_{I\hat{I}}$ the set of those who are inactive in both allocations, by $S_{A\hat{I}}$ the set of those who are active for the initial allocation and inactive for the alternative allocation, by $S_{I\hat{A}}$ the set of those who are inactive for the initial allocation and active for the alternative allocation. By definition one gets:

$$\int_{S_{A\hat{A}}} \left(\int_{\mathbb{R}} \hat{c}(y,z,x)dG(x)\right) h(y,z)dydz + \int_{S_{I\hat{I}}} \hat{c}(y,z)h(y,z)dydz \geq \int_{S_{A\hat{A}}\cup S_{I\hat{I}}} c(y,z)h(y,z)dydz$$

$$\int_{S_{A\hat{I}}} [\hat{c}(y,z)+z] h(y,z)dydz \geq \int_{S_{A\hat{I}}} c(y,z)h(y,z)dydz$$

$$\int_{S_{I\hat{A}}} \left(\int_{\mathbb{R}} \hat{c}(y,z,x)dG(x)\right) h(y,z)dydz \geq \int_{S_{I\hat{A}}} [c(y,z)+z] h(y,z)dydz,$$

with some strict inequality. Summing up the three previous equations yields

$$\int_{S_{\hat{A}}} \left(\int_{\mathbb{R}} \hat{c}(y,z,x)dG(x) \right) h(y,z)dydz + \int_{S_{\hat{I}}} \hat{c}(y,z)h(y,z)dydz >$$
$$\int_{S} c(y,z)h(y,z)dydz + \int_{S_{I\hat{A}}} zh(y,z)dydz - \int_{S_{A\hat{I}}} zh(y,z)dydy. \tag{A1}$$

From condition *3* it follows that

$$\int_{S_{I\hat{A}}} zh(y,z)dydz \geq \int_{S_{I\hat{A}}} Y^{*}(y)h(y,z)dydz$$

$$\int_{S_{A\hat{I}}} zh(y,z)dydz \leq \int_{S_{A\hat{I}}} Y^{*}(y)h(y,z)dydz.$$

These two equations imply, together with (A1):

$$\int_{S_{\hat{A}}} \left(\int_{\mathbb{R}} \hat{c}(y,z,x)dG(x) \right) h(y,z)dydz + \int_{S_{\hat{I}}} \hat{c}(y,z)h(y,z)dydz >$$
$$\int_{S} c(y,z)h(y,z)dydz + \int_{S_{I\hat{A}}} Y^{*}(y)h(y,z)dydz - \int_{S_{A\hat{I}}} Y^{*}(y)h(y,z)dydz.$$

As $c(y,z)$ is feasible, it satisfies

$$\int_{S} c(y,z)h(y,z)dydz = \int_{S_{A\hat{I}} \cup S_{A\hat{A}}} Y^{*}(y)h(y,z)dydz,$$

which yields

$$\int_{S_{\hat{A}}} \left(\int_{\mathbb{R}} \hat{c}(y,z,x)dG(x) \right) h(y,z)dydz + \int_{S_{\hat{I}}} \hat{c}(y,z)h(y,z)dydz >$$
$$\int_{S_{A\hat{I}} \cup S_{A\hat{A}}} Y^{*}(y)h(y,z)dydz + \int_{S_{I\hat{A}}} Y^{*}(y)h(y,z)dydz - \int_{S_{A\hat{I}}} Y^{*}(y)h(y,z)dydz =$$
$$\int_{S_{I\hat{A}} \cup S_{A\hat{A}}} Y^{*}(y)h(y,z)dydz.$$

As $S_{\hat{A}} = S_{A\hat{A}} \cup S_{A\hat{I}}$, one gets:

$$\int_{S_{\hat{A}}} \left(\int_{\mathbb{R}} \hat{c}(y,z,x)dG(x) \right) h(y,z)dydz + \int_{S_{\hat{I}}} \hat{c}(y,z)h(y,z)dydz > \int_{S_{\hat{A}}} Y^{*}(y)h(y,z)dydz.$$

From the productive efficiency condition *1* one has $Y^{*}(y) \geq Y(y), \forall y$. This condition implies, together with the previous inequality:

$$\int_{S_{\hat{A}}} \left(\int_{\mathbb{R}} \hat{c}(y,z,x)dG(x) \right) h(y,z)dydz + \int_{S_{\hat{I}}} \hat{c}(y,z)h(y,z)dydz > \int_{S_{\hat{A}}} Y(y)h(y,z)dydz,$$

which proves that the alternative allocation is not feasible. ∎

B   Proof of Proposition 3

Let us first notice that when the state implements the tax-subsidy scheme $\{b(w), \tau(w), f(w), \rho\}$, the equilibrium wage is an increasing function of y that is denoted by $w(y)$. The equilibrium values of the other variables can be denoted as $b(y) = b(w(y))$, $\tau(y) = \tau(w(y))$, $f(y) = f(w(y))$, $X(y) = X(w(y), y)$, $q(y) = G(X(y))$ and $Z(y) = Z(w(y), y)$.

The part A) of Proposition 3 is proved as follows. First, we define the optimal value of $\{w(y), b(y), X(y), \rho\}$ for any second-best financial incentives to work $\tilde{Z}(y) \leq Y^*(y)$. Then we find out how this solution can be implemented by the appropriate choice of $\{\tau(w), b(w), f(w), \rho\}$.

The budget constraint of the state reads

$$\int_{y_{\min}}^{+\infty} \left(\int_{-\infty}^{Z(y)} \{[1 - G(X(y))] \tau(y) + G(X(y)) [f(y) - b(y)]\} h(y, z) dz \right) dy \geq \rho \left[1 - \int_{y_{\min}}^{+\infty} H[y, Z(y)] dy \right],$$
(B1)

where $H[y, Z(y)] = \int_{-\infty}^{Z(y)} h(y, z) dz$. Using the free entry condition:

$$\int_{X(y)}^{+\infty} [x \cdot y - w(y) - \tau(y)] dG(x) - G(X(y)) f(y) = k(y), \quad \forall y \geq y_{\min},$$
(B2)

the budget constraint of the state (B1) can be rewritten as:

$$\int_{y_{\min}}^{+\infty} \{Y(y) - [1 - G(X(y))] w(y) - G(X(y)) b(y)\} H[y, Z(y)] dy \geq \rho \left[1 - \int_{y_{\min}}^{+\infty} H[y, Z(y)] dy \right]$$
(B3)

where $Y(y) = y \int_{X(y)}^{+\infty} x dG(x) - k(y)$.

Accordingly, the maximization problem which defines the optimal value of $\{w(y), b(y), X(y), \rho\}$ for any second-best financial incentives to work $\tilde{Z}(y) \leq Y^*(y)$ reads

$$\max_{\{w(y), b(y), X(y), \rho\}} \rho$$

subject to

$$v\left[\tilde{Z}(y) + \rho\right] = [1 - G(X(y))] v(w(y)) + G(X(y)) v(b(y)), \quad \forall y \geq y_{\min}$$
(B4)

$$\int_{y_{\min}}^{+\infty} \{Y(y) - [1 - G(X(y))] w(y) - G(X(y)) b(y)\} H\left[y, \tilde{Z}(y)\right] dy \geq \rho \left[1 - \int_{y_{\min}}^{+\infty} H\left[y, \tilde{Z}(y)\right] dy \right].$$
(B5)

Let us denote by $\lambda(y)$ and μ the Lagrange multipliers associated with constraints (B4) and (B5) respectively. The Lagrangian reads

$$\mathcal{L} = \rho + \int_{y_{\min}}^{+\infty} \lambda(y) \left\{ [1 - G(X(y))] v(w(y)) + G(X(y)) v(b(y)) - v\left[\tilde{Z}(y) + \rho\right] \right\} dy +$$

$$\mu \left[\int_{y_{\min}}^{+\infty} \{Y(y) - [1 - G(X(y))] w(y) - G(X(y)) b(y)\} H\left[y, \tilde{Z}(y)\right] dy - \rho \left[1 - \int_{y_{\min}}^{+\infty} H\left[y, \tilde{Z}(y)\right] dy \right] \right]$$

The first-order conditions can be written as

$$\frac{\partial \mathcal{L}}{\partial X(y)} = 0 \Leftrightarrow \lambda(y)\left[v(b(y)) - v(w(y))\right] = \mu\left(yX(y) - [w(y) - b(y)]\right) H\left[y, \tilde{Z}(y)\right], \quad \forall y \geq y_{\min}, \quad \text{(B6)}$$

$$\frac{\partial \mathcal{L}}{\partial w(y)} = 0 \Leftrightarrow \lambda(y)v'(w(y)) = \mu H\left[y, \tilde{Z}(y)\right], \quad \forall y \geq y_{\min}, \quad \text{(B7)}$$

$$\frac{\partial \mathcal{L}}{\partial b(y)} = 0 \Leftrightarrow \lambda(y)v'(b(y)) = \mu H\left[y, \tilde{Z}(y)\right], \quad \forall y \geq y_{\min}, \quad \text{(B8)}$$

$$\frac{\partial \mathcal{L}}{\partial \rho} = 0 \Leftrightarrow 1 - \int_{y_{\min}}^{+\infty} \lambda(y)v'\left[\tilde{Z}(y) + \rho\right] dy = \mu\left[1 - \int_{y_{\min}}^{+\infty} H\left[y, \tilde{Z}(y)\right] dy\right]. \quad \text{(B9)}$$

Equations (B7), (B8) and (B4) imply that $b(y) = w(y) = \tilde{Z}(y) + \rho$, $\forall y \geq y_{\min}$. As $w(y) = \tilde{Z}(y) + \rho$, equation (B7) reads $\lambda(y)v'(\tilde{Z}(y) + \rho) = \mu H\left[y, \tilde{Z}(y)\right]$ which yields in (B9): $\mu = 1$. Thus, equation (B7) implies that $\lambda(y) > 0, \forall y \geq y_{\min}$. Eventually, when $b(y) = w(y), \lambda(y) > 0, \forall y \geq y_{\min}$ and $\mu > 0$, equation (B6) implies that $X(y) = 0$.

At this stage, it has been proved that the optimal value of $\{w(y), b(y), X(y), \rho\}$ satisfies $b(y) = w(y)$ and $X(y) = 0$ for any second-best financial incentives to work $\tilde{Z}(y) \leq Y^*(y)$. These properties transform our model into a particular version of Laroque's (2005) model of labor supply decisions at the extensive margin with no unemployment. Our model is now such that a type-(y, z) agent who decides to "work" produces $Y^*(y)$ and earns an income equal to $\tilde{Z}(y) + \rho$; if he decides to stay idle he produces nothing and earns $z + \rho$. Theorem 3 in Laroque (2005) completely characterizes the second-best optimal financial incentives to work $\tilde{Z}(y)$ in this case. It is shown that feasible financial incentives to work $\tilde{Z}(y)$, such that $\tilde{Z}(y) \leq Y^*(y)$, support a second-best allocation if and only if no category of ability y is overtaxed at $\tilde{Z}(y)$. This result can be understood more precisely by looking at the relation between the budget of the state and the financial incentives to work. The net surplus that the state gets from individuals of ability y, denoted by $B(y)$, reads:

$$B(y) = \int_{-\infty}^{Z(y)} \left\{G(0)\left[f(w(y)) - b(w(y))\right] + [1 - G(0)]\tau(w(y))\right\} h(y, z)dz - \rho \int_{Z(y)}^{+\infty} h(y, z)dz$$

With the help of (9) and (10), one knows that $f(w) - b(w) = \tau(w)$, hence:

$$B(y) = \int_{-\infty}^{Z(y)} \tau(w(y))h(y, z)dz - \rho \int_{Z(y)}^{+\infty} h(y, z)dz$$

Using equations (4), (9) and (10), the free entry condition $J(w, y) = 0$ implies that taxes levied on an employed worker of ability y are equal to $Y^*(y) - w(y)$. When active individuals are perfectly insured, equations (7) and (9) imply that $Z(y) = w(y) - \rho$. Therefore, $\tau(w(y))$, the taxes levied on an employed worker of ability y are equal to $Y^*(y) - Z(y) - \rho$, and $B(y)$ reads:

$$B(y) = \left[Y^*(y) - Z(y)\right] H\left[y, Z(y)\right] - \rho \int_{-\infty}^{+\infty} h(y, z)dz,$$

where $H(y, Z) = \int_{-\infty}^{Z} h(y, z) dz$ denotes the distribution of the tastes for leisure conditional on the ability y of individuals, or, in other words, the labor force participation rate of the individuals with ability y when the financial incentives to work amount to Z. This relation shows that $B(y)$ is equal to the production of active individuals minus their financial incentives to work, minus the cost of the income guarantee.

In this context, the set of agents of ability y is overtaxed at financial incentives to work $Z(y)$ if there exists some $Z > Z(y)$ such that $[Y^*(y) - Z] H(y, Z) \geq [Y^*(y) - Z(y)] H[y, Z(y)]$. If agents of ability y are overtaxed at $Z(y)$, the state can provide them a higher level of utility, equal to $\max[v(Z + \rho), v(z + \rho)]$, with at least the same income $[Y^*(y) - Z] H(y, Z)$. Accordingly, overtaxation cannot be optimal.

The optimal value of ρ can be obtained by substituting the values of $w(y)$ and $b(y)$ which are equal to $\tilde{Z}(y) + \rho$ into the – binding – constraint (B5). One gets $\rho = \int_{y_{\min}}^{+\infty} \left[Y^*(y) - \tilde{Z}(y)\right] H\left[y, \tilde{Z}(y)\right] dy$. Let us find out how this solution can be implemented by the appropriate choice of $\{\tau(w), b(w), f(w), \rho\}$.

The equality $b(y) = w(y)$ is merely implemented by $b(w) = w$ which proves condition *1.* of part A) of Proposition 3.

The appropriate choice of $\tau(w)$ and $f(w)$ can be defined by noticing that there exists a bijection between $(\tau(y), f(y))$ and $(w(y), X(y))$ which is defined by two equations: namely the reservation productivity of the firms (equation (4))

$$X(y) = \frac{w(y) + \tau(y) - f(y)}{y}, \quad \forall y \geq y_{\min}, \tag{B10}$$

and the free entry condition (B2), which reads, using the definition of the reservation productivity of the firms (B10):

$$f(y) = -k(y) + \int_{X(y)}^{+\infty} [x \cdot y - X(y) \cdot y] \, dG(x), \quad \forall y \geq y_{\min}. \tag{B11}$$

The definition (B10) of the reservation productivity of the firms implies, together with $w(y) = b(y)$, that $X(y) = 0$ is implemented by:

$$f(w) = b(w) + \tau(w),$$

which proves condition *2.* of part A) of Proposition 3.

When $X(y) = 0$, equation (B11) implies that $f(y) = Y^*(y)$. The function $\tilde{Z}(y)$ being strictly increasing with y, the equality $\tilde{Z}(y) = w(y) - \rho$ can be written as $y = \tilde{Z}^{-1}(w(y) - \rho)$, which defines, together with $Y^*(y) = f(y)$, the function $f(w)$ that reads:

$$f(w) = Y^* \left[\tilde{Z}^{-1}(w - \rho)\right] \quad \text{with} \quad \rho = \int_{y_{\min}}^{+\infty} \left[Y^*(y) - \tilde{Z}(y)\right] H\left[y, \tilde{Z}(y)\right] dy.$$

This proves claim *3.* of part A) of Proposition 3.

At this stage, it remains to prove the part B) of Proposition 3 that exhibits additional assumptions on the functions $\tilde{Z}(y)$ and $Y^*(y)$ in order to satisfy the Assumptions 1 and 2 necessary for the existence and uniqueness of the decentralized equilibrium. Let us first consider Assumption 2.

When the optimal tax-subsidy scheme is implemented, equation (5) and condition 2. of part A) of Proposition 3, give:

$$J(w,y) = Y^*(y) - f(w) = Y^*(y) - Y^* \left[\tilde{Z}^{-1}(w - \rho) \right] \tag{B12}$$

As it has been assumed that the function $k(y)$ is continuous in y, it follows that $Y^*(y) = y \int_0^{+\infty} x dG(x) - k(y)$ is also continuous in y. If we suppose that $\tilde{Z}$ is continuous, then $\tilde{Z}^{-1}$ is also continuous, and so is $Y^* \left[\tilde{Z}^{-1}(w - \rho) \right]$. Thus, $J(w,y)$ is a continuous function of w, and condition 2.i of Assumption 2 is satisfied. If one assumes that $Y^*(y)$ is stricly increasing with y, equation (B12) shows that $J(w,y)$ is also strictly increasing with y. Condition 2.ii of Assumption 2 is thus satisfied.

Let us now consider Assumption 1. For any y (B12) shows that $J(w(y),y) = 0$ when $w(y) = \tilde{Z}(y) + \rho$. Thus, condition 1.i of Assumption 1 is satisfied. With the help of (B12), one sees that condition 1.ii of Assumption 1 is equivalent to:

$$Y^*(y) < \lim_{w \to +\infty} Y^* \left[\tilde{Z}^{-1}(w - \rho) \right], \ \forall y$$

The function $Y^*(y)$ being strictly increasing in y, this inequality is equivalent to

$$y < \lim_{w \to +\infty} (Y^*)^{-1} \left\{ Y^* \left[\tilde{Z}^{-1}(w - \rho) \right] \right\} = \lim_{w \to +\infty} \left[\tilde{Z}^{-1}(w - \rho) \right], \ \forall y$$

The function $\tilde{Z}$ being strictly increasing, this inequality is thus equivalent to

$$\tilde{Z}(y) < \lim_{w \to +\infty} \tilde{Z} \left[\tilde{Z}^{-1}(w - \rho) \right] = \lim_{w \to +\infty} (w - \rho) = +\infty, \ \forall y$$

Hence, the condition 1.ii of Assumption 1 is satisfied when the function $\tilde{Z}(y)$ is bounded. ∎

C   The case of equal valuation of leisure when unemployed and when inactive

Let us consider the extreme assumption where individuals value their leisure time equally when they are laid off and when they are inactive. If the unemployed workers get $b + z$ instead of b, individuals decide to participate in the labor market if

$$[1 - q(w,y)] \max[v(w), v(b(w) + z)] + q(w,y)v \left[b(w) + z \right] \geq v(z + \rho) \tag{C13}$$

In the case where $v(w) \geq v(b(w) + z)$, this last relation is equivalent to:

$$[1 - q(w,y)] v(w) \geq v(z + \rho) - q(w,y)v \left[b(w) + z \right]$$

Let us consider the function $\phi(z) = v(z + \rho) - q(w,y)v[b(w) + z]$. One gets $\phi'(z) = v'(z + \rho) - q(w,y)v'[b(w) + z]$, since $v'' < 0$, when $b(w) \geq \rho$ we have $v'(z + \rho) \geq v'(b(w) + \rho) \geq q(w,y)v'[b(w) + z]$ and then $\phi'(z) \geq 0$. Therefore, when $v(w) \geq v(b(w) + z)$, the condition (C13) is satisfied for $z \leq Z(w,y)$ such that,

$$[1 - q(w,y)]\, v(w) + q(w,y)v[b(w) + Z(w,y)] = v(Z(w,y) + \rho)$$

This last equality defines $Z(w,y)$ and can be subsituted to the equation (7) of the paper.

The maximization problem which defines the optimal value of $\{w(y), b(y), X(y), \rho\}$ for any second-best financial incentives to work $\tilde{Z}(y) \leq Y^*(y)$ now reads

$$\max_{\{w(y),b(y),X(y),\rho\}} \rho$$

subject to

$$v\left[\tilde{Z}(y) + \rho\right] = [1 - G(X(y))]\, v\,(w(y)) + G(X(y))v(b(y) + \tilde{Z}(y)), \quad \forall y \geq y_{\min} \qquad \text{(C14)}$$

$$\int_{y_{\min}}^{+\infty} \{Y(y) - [1 - G(X(y))]\, w(y) - G(X(y))b(y)\}\, H\left[y, \tilde{Z}(y)\right] dy \geq \rho\left[1 - \int_{y_{\min}}^{+\infty} H\left[y, \tilde{Z}(y)\right] dy\right].$$
$$\text{(C15)}$$

Let us denote by $\lambda(y)$ and μ the Lagrange multipliers associated with constraints (C14) and (C15) respectively. The Lagrangian reads

$$\mathcal{L} = \rho + \int_{y_{\min}}^{+\infty} \lambda(y) \left\{ [1 - G(X(y))]\, v\,(w(y)) + G(X(y))v(b(y) + \tilde{Z}(y)) - v\left[\tilde{Z}(y) + \rho\right] \right\} dy +$$
$$\mu\left[\int_{y_{\min}}^{+\infty} \{Y(y) - [1 - G(X(y))]\, w(y) - G(X(y))b(y)\}\, H\left[y, \tilde{Z}(y)\right] dy - \rho\left[1 - \int_{y_{\min}}^{+\infty} H\left[y, \tilde{Z}(y)\right] dy\right]\right]$$

The first-order conditions can be written as

$$\frac{\partial \mathcal{L}}{\partial X(y)} = 0 \Leftrightarrow \lambda(y)\left[v(b(y) + \tilde{Z}(y)) - v(w(y))\right] = \mu\,(yX(y) - [w(y) - b(y)])\, H\left[y, \tilde{Z}(y)\right], \quad \forall y \geq y_{\min},$$
$$\text{(C16)}$$

$$\frac{\partial \mathcal{L}}{\partial w(y)} = 0 \Leftrightarrow \lambda(y)v'(w(y)) = \mu H\left[y, \tilde{Z}(y)\right], \quad \forall y \geq y_{\min}, \qquad \text{(C17)}$$

$$\frac{\partial \mathcal{L}}{\partial b(y)} = 0 \Leftrightarrow \lambda(y)v'(b(y) + \tilde{Z}(y)) = \mu H\left[y, \tilde{Z}(y)\right], \quad \forall y \geq y_{\min}, \qquad \text{(C18)}$$

$$\frac{\partial \mathcal{L}}{\partial \rho} = 0 \Leftrightarrow 1 - \int_{y_{\min}}^{+\infty} \lambda(y)v'\left[\tilde{Z}(y) + \rho\right] dy = \mu\left[1 - \int_{y_{\min}}^{+\infty} H\left[y, \tilde{Z}(y)\right] dy\right].$$

Equations (C17), (C18) and (C14) imply that $b(y) + \tilde{Z}(y) = w(y) = \tilde{Z}(y) + \rho$, $\forall y \geq y_{\min}$. Thus $b(y) = \rho$. Equation (C16) yields $yX(y) - [w(y) - b(y)] = 0$ and thus

$$X(y) = \frac{\tilde{Z}(y)}{y}$$

As we still have

$$X(y) = \frac{w(y) + \tau(w(y)) - f(w(y))}{y}$$

it appears that $\tilde{Z}(y) = w(y) + \tau(w(y)) - f(w(y))$, and then $w(y) - b(w(y)) = w(y) + \tau(w(y)) - f(w(y))$.

Therefore, we get

$$f(w) = b(w) + \tau(w) = \rho + \tau(w).\blacksquare$$